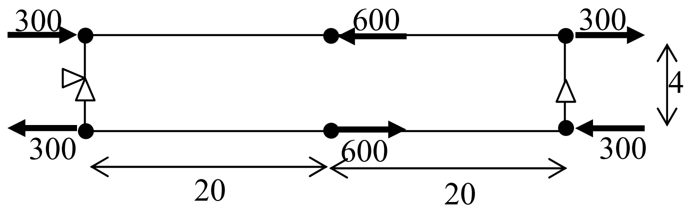
Mini-FEA Assignment 4.1.2

Comparison of FEA and Beam Theory Predictions For Beam with Multiple Couples

Geometry: Length = 40, Height = 4 Material: $E = 10^6$ (1E6), $\nu = 0.4$.
Mesh: 20 x 2 Quadratic Elements.
Loads: at (x,y) = (0,2), Ux = 0, Uy = 0; at (40,2), Fx = 0, Uy = 0; all other forces are zero except at nodes shown (Fy = 0 at nodes shown, Fx has indicated values).



FEA Results to Extract

- σ_x and ε_x at (x,y) = (2,0), (2,4), (10,0), (10,4), (30,0), (30,4).
- Ux at (x,y) = (7,4), (9,4), (7,3), (9,3), (7,2), (9,2), (7,1), (9,1), (7,0), (9,0).
- Uy at points on the centerline (x,y) = (7,2), (8,2) and (9,2).

Analyses to Compare with FEA Results

(i) Stress Comparison

- Convert three pairs of equal and opposite forces to three couples
- For beam with these three couples applied, draw the shear force (V) and bending moment (M) diagrams for this beam.
- At cross-sections, x = 2, 10, and 30, calculate bending stress at top and bottom, using bending stress (beam theory) formula $\sigma = -My/I$. Remember that in this bending stress formula y is zero at the centroid, not the bottom; also be careful about signs.
- Compare your calculated bending stress with stress from FEA.
- For each of the three cross-sections, note whether stresses from FEA and beam theory agree reasonably well (within a few percent). If they do not, discuss why not.

(ii) Shape Change

- Draw original rectangular boundary, and on top of it draw deformed boundary.
- Observe how square formed by points (8,0), (12,0), (8,4) and (12,4) changes as the mesh is deformed.
- Observe how square formed by points (28,0), (32,0), (28,4) and (32,4) changes.
- Draw original and deformed shape of two squares by hand. Underneath, write down bending moment (M) at these two cross-sections.

(iii) Strain related to displacements

- Use Ux from FEA to calculate the change in length of the five horizontal segments (7 to 9,0), (7 to 9,1), (7 to 9,2), (7 to 9,3), and (7 to 9,4). (remember: $\delta = U_{right} - U_{left}$)
- Calculate the axial strain of these segments from $\varepsilon = \delta/L$ and enter into table.
- Enter into table FEA strains ε_x and strains from beam theory $\varepsilon = -My/EI$ at points (8,0), (8,1), (8,2), (8,3), and (8,4), midpoints of above five segments.

(iv) Deflection, Slope, Curvature

- Estimate the slope of centerline at $x = 7.5$, using $\Delta U_y/\Delta x$, with U_y at (7,2) and (8,2).
- Estimate the slope of centerline at $x = 8.5$, using $\Delta U_y/\Delta x$, with U_y at (8,2) and (9,2).
- Estimate curvature, $\kappa = d^2U_y/dx^2$ from $\Delta(\text{slope})/\Delta x$, using slopes at $x = 8.5$ and 9.5.
- Calculate $M = EI\kappa$, and compare this with bending moment M at $x = 8$ found earlier.

Results

(i) Stress Comparison

Draw body as a beam with loads. Underneath, draw V and M diagrams (watch signs).

Show terms in calculation of stress from beam theory $\sigma = -My/I$.

	(2,0)	(2,4)	(10,0)	(10,4)	(30,0)	(30,4)
σ_x (FEA)						
σ_x (Beam)						

Comment on stress comparison:

(ii) Shape Change
Draw original shape of rectangular boundary. Draw on top of it the shape of the boundary when the body is deformed.

Draw original shape of two squares then as they are deformed. Write down bending moment (with sign) underneath each.

(iii) Strain related to displacements

	(7,4)	(9,4)	(7,3)	(9,3)	(7,2)	(9,2)	(7,1)	(9,1)	(7,0)	(9,0)
Ux										

	(8,4)	(8,3)	(8,2)	(8,1)	(8,0)
ε_x (δ/L)					
ε_x (FEA)					
ε_x (Beam)					

(iv) Deflection, Slope, Curvature

	(7,2)	(8,2)	(9,2)
Uy			

	(7.5,2)	(8.5,2)
Slope (= $\Delta U_y/\Delta x$)		

	$\kappa = \Delta(\text{Slope})/\Delta x$	$M = EI\kappa$	M (from bending)
At x = 8			