A STATICS CONCEPT INVENTORY: DEVELOPMENT AND PSYCHOMETRIC ANALYSIS

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ABSTRACT

Quantification of conceptual understanding of students in Statics has been undertaken. Drawing on a prior study identifying the fundamental concepts and typical student errors in Statics, multiple choice questions have been devised to probe students' ability to use concepts in isolation. This paper describes a testing instrument comprising such questions, as well as psychometric analyses of test results of 245 students at five universities.

INTRODUCTION

It is increasingly appreciated that learning is tied to effective assessment: monitoring student progress and feeding that information back to students [1]. There are many aspects of learning that can be assessed. However, if we seek to empower students to transfer the knowledge gained to new situations, then a deep understanding must be developed [2]. In many engineering science courses, deep understanding is usually associated with understanding of concepts. Thus, some effort must be devoted to identifying core concepts and then to devising means of gauging students understanding of those concepts. This paper describes efforts to measure student understanding of concepts in Statics, and to judge the effectiveness of the resulting testing instrument.

Engineering Statics is a subject that is extremely worthy of this heightened level of attention. Statics is a pivotal course in a several engineering disciplines, preparing students for a number of follow-on courses, such as dynamics, mechanics of materials, and, ultimately, design. Instructors of these follow-on courses, as well as instructors of engineering design, often feel that student understanding of Statics is a major impediment to their success in these courses. At the same time, instructors are seeking to improve instruction in Statics. Judging such instructional innovations should, at least in part, be based on their ability to advance student understanding as captured by clear, agreed upon measures. Thus assessment of conceptual understanding can help instructors to gauge the effectiveness of new teaching methods and approaches.

In the case of Newtonian mechanics, there have been efforts by the physics education community [3,4] to identify its basic concepts and associated misconceptions. These have lead to the development of instruments for measuring conceptual understanding in physics [5]. With the force Concept Inventory (of Newtonian mechanics) as a model, there also have been recent efforts in the engineering education community to develop concept inventories for a variety of engineering subjects [6], including preliminary efforts in Statics [7,8].

Little work has been devoted to identifying student misconceptions in Statics specifically. This paper draws upon the first author's recent effort to establish a conceptual framework for Statics [9]. Four basic concept clusters were proposed. The most common errors of students, identified through collection and analysis of student work, were identified, and these errors were explained on the basis of inadequacies in student understanding of the concept clusters. This paper will show how understanding of many of these concepts can be gauged through multiple choice questions [10]. Results of administering a test composed of such questions to 245 students at five universities will be presented. Conclusions regarding the psychometric effectiveness of the test and implications for Statics instruction will be drawn.

CONCEPTS OF STATICS

One class of Statics problems that is directly relevant to engineering systems involves the analysis of multiple, connected bodies. The conceptual framework described by Steif [9] was devised with this class of problems in mind, and it consists of four clusters of concepts as follows:

C1. Forces are always in equal and opposite pairs acting *between* bodies, which are usually in contact.

C2. Distinctions must be drawn between a force, a moment due to a force about a point, and a couple. Two combinations of forces and couples are statically equivalent to one another if they have the same net force and moment.

C3. The possibilities of forces between bodies that are connected to, or contact, one another can be reduced by virtue of the bodies themselves, the geometry of the connection and/or assumptions on friction.

C4. Equilibrium conditions always pertain to the external forces acting directly on a chosen body, and a body is in equilibrium if the summation of forces on it is zero and the summation of moments on it is zero.

Solving problems in Statics involves reasoning about physical systems, translating the interactions between parts of systems into the symbols and variables of Statics, and then deriving meaningful relations between the variables based on the principle of equilibrium. The concepts just described pertain primarily to the modeling steps of translating features of the system into symbols and variables. There are clearly also important skills associated with carrying out mathematical operations, such as resolving or combining forces and finding moments due to forces. There are also less acknowledged skills that involve reasoning about physical systems: recognizing the distinct parts making up a mechanical system and discerning how they are connected to one another. Thus, understanding of concepts outlined above is critical to problem solving in Statics, but additional skills are relevant as well.

CONCEPTUAL ERRORS IN STATICS

Certain types of errors that students make in solving Statics problems recur with great frequency. Based on observations of students' work, and the sharing of experiences between instructors at various institutions, these errors have been identified and organized into categories [9]. Expressed succinctly, these errors are:

1. Failure to be clear as to which body is being considered for equilibrium

2. Failure to take advantage of the options of treating a collection of parts as a single body, dismembering a system into individual parts, or dividing a part into two

3. Leaving a force off the free body diagram (FBD) when it should be acting

4. Drawing a force as acting on the body in the FBD, even though that force is exerted by a part which is also included in the FBD

5. Drawing a force as acting on the body of the FBD, even though that force does not act directly on the body

6. Failing to account for the mutual (equal and opposite) nature of forces between connected bodies that are separated for analysis

7. Ignoring a couple that could act between two bodies or falsely presuming its presence

8. Not allowing for the full range of possible forces between connected bodies, or not sufficiently restricting the possible forces

9. Presuming a friction force is at the slipping limit (μN), even though equilibrium is maintained with a friction force of lesser magnitude

10. Failure to impose balance of forces in all directions and moments about all axes

11. Having a couple contribute to a force summation or improperly accounting for a couple in a moment summation

GAUGING CONCEPTUAL UNDERSTANDING WITH MULTIPLE CHOICE QUESTIONS

To identify specific conceptual lapses, we have devised questions that focus rather narrowly on particular concepts in isolation. In each question students select from five choices; this allows for simple quantification of performance. In addition, the wrong answers to questions are specifically chosen to reflect known conceptual errors exhibited by students, such as those outlined above. As these questions are intended to detect errors reflecting incorrect concepts, rather than errors in mathematical analysis, most questions do not involve computation. For those questions that involve computation, the computations are extremely simple. Since each wrong answer represents a correct computation based on an incorrect conception, and the computations themselves are trivial, such questions allow us to detect conceptual misunderstanding. Such a question is shown below as the friction sample.

The five classes of questions are as follows:

- *Free body diagrams*

These questions capture a combination of concept cluster C1 on the inter-body nature of forces and the first half of cluster C4, namely, that equilibrium always pertains to a body. In these questions, students must think about the forces that act on subsets of a system. There are no complications associated with the direction of forces, and any use of equilibrium is trivial (for example, summation of forces in a single direction). Errors 3, 4, 5, and to a lesser extent 1 are at issue.

- *Static equivalence of combinations of forces and couples*

In these questions, which capture concept cluster C2, students must be able to determine whether one combination of forces and couples can be replaced with another combination and still maintain equilibrium. There is no issue of what forces and couples are actually exerted by contacting bodies, but only the equivalence between sets of vectors. However, the calculations are trivial; thus, the focus is on understanding the distinctions between force, moment and couple and their inter-relations Errors 10 and 11 are at issue.

- *Type and direction of loads at connections (including different situations of roller, pin in slot, general pin joint, and pin joint on a two-force member)*

These questions capture one aspect of concept cluster C3, namely the simplifications in the forces between connected bodies when the usual assumption of negligible friction is made. Students must recognize the implications of the joint regarding direction of force, and not be swayed by directions of applied forces or orientation of members. Errors 7 and 8 are at issue.

- *Limit on the friction force and its trade-off with equilibrium conditions*

These questions capture a second aspect of concept cluster C3, namely reasoning about the forces between stationary contacting bodies when the force at which slip occurs is described by Coulomb friction. Error 9 is at issue.

- *Equilibrium conditions*

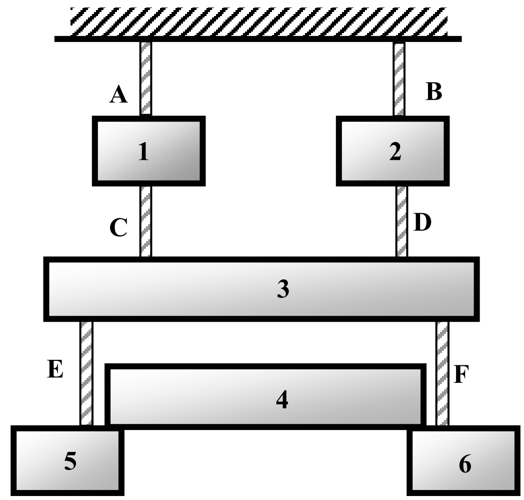
These questions capture the second portion of concept cluster C4, namely the necessity for both forces and moments acting on a body to sum to zero. Errors 10 and 11 are at issue.

SAMPLE CONCEPTUAL QUESTIONS

Here we show samples of questions devised to test conceptual understanding of free body diagrams, static equivalence, forces at connections, friction forces and equilibrium.

<u>Sample question on free body diagrams</u>

Consider the configuration shown. A free body diagram is to be constructed which includes blocks 2 and 3 and the cord connecting them.



Which is the correct free body diagram?

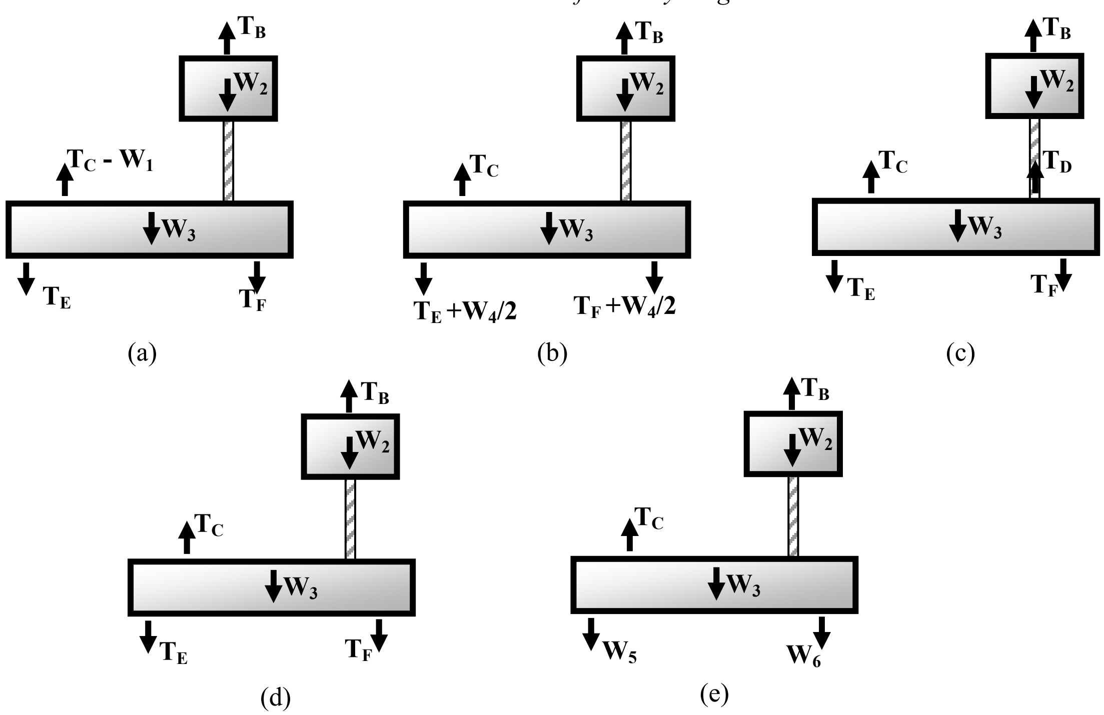


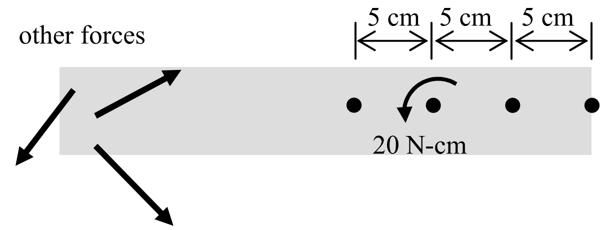
Figure 1. Example of concept question addressing static equivalency

Notice that besides the correct response (d), the other responses capture various types of generic errors. Options (a) and (b) are two versions of the error of including a force which does not act on the body being isolated. The operating force is the tension of the attached cable (T_C, T_E, or T_F). By contrast, the weight, which is subtracted in case (a) and added in (b), is a force between the earth and a block not in the diagram, and should not be included. Choosing option (e) probably signals a false equivalence between the rope tension and the weight W_5, stemming from failure to include the weight W_4 which also is supported by the tension in cable E. Option (c), which is widely chosen, reflects the failure to reject forces which act between bodies, both of which are included in the free body diagram.

From previous experience with variants on such questions, when one option had a force obviously missing, students apparently recognized it was wrong, perhaps by comparison with the alternatives, and rarely chose it. Also, to address the issue of equal and opposite forces between contacting bodies (Newton's 3rd law), questions were tried which required free body diagrams of two connected subsystems; those options which failed to satisfy Newton's 3rd law apparently were rarely chosen. Thus, while students often make those mistakes in solving problems, we were not successful in devising multiple choice questions that ferreted out those misconceptions. New versions of such conceptual tests currently being contemplated, which do not provide the crutch of elimination by comparison, may be able to uncover such misconceptions.

Sample question on static equivalence

One couple of magnitude 20 N-cm keeps the member in equilibrium while it is subjected to other forces acting in the plane at various points (shown at the left). The four dots denote equally spaced points along the member.



Assuming that the same forces are applied at the left, what load(s) could replace the 20 N-cm couple and still maintain equilibrium?

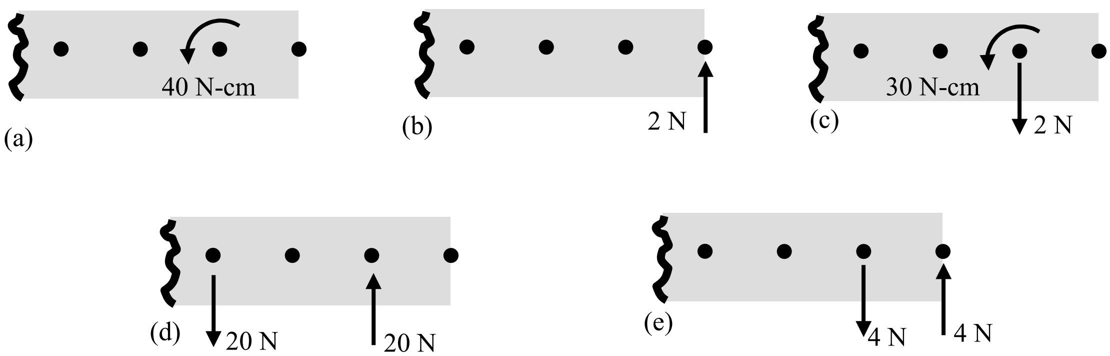
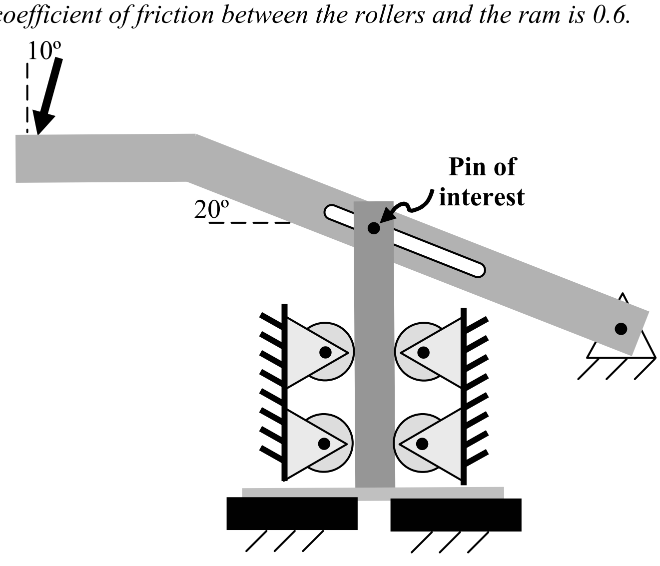


Figure 2. Example of concept question addressing static equivalence

Option (a) captures the misconception that moving a couple changes the moment that it exerts. Option (b) captures the misconception that a force can be equivalent to a couple, if it provides the right moment about a point. Option (c) makes the impossible presence of a force apparently more palatable by including a couple as well (that again produces the correct moment about the original point). Option (d) while appropriately leading to zero net force produces the wrong net moment (the distance between the forces is ignored). The correct option (e) is made slightly more difficult to choose because the pair of forces is not centered about the point where the original couple is applied, although this makes no difference statically.

Sample question on simplification of forces between connected bodies

The mechanism is acted upon by the force shown acting at 10º. It drives the vertical ram which punches the sheet. The coefficient of friction between the rollers and the ram is 0.6.



What is the direction of the force exerted by the slot on the pin of interest? (All pins have been identified as frictionless).

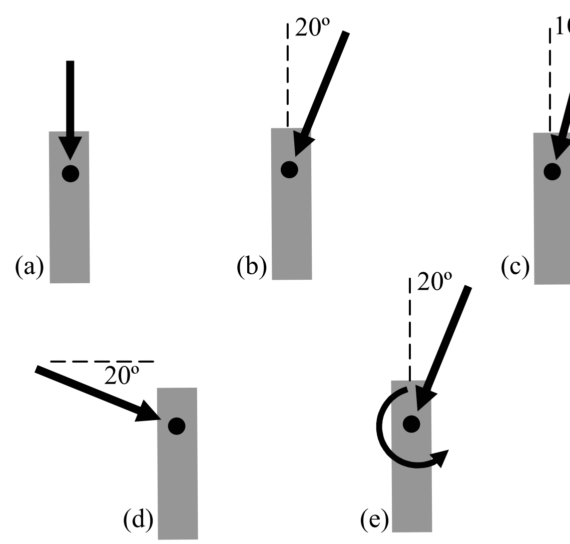
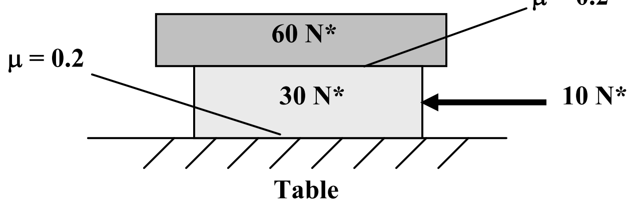


Figure 3. Example of concept question addressing simplification of forces between contacting bodies.

This problem is based on the recognition that the force associated with the frictionless pin contacting the surface of the slot must act perpendicularly to the slot, irrespective of any other forces acting or of the orientation of any members. Option (a) tempts students with the incorrect possibility that the force acts parallel to the member on which it acts (a common assumption, perhaps tied to indiscriminant application of the result for a two-force body). Option (b) is correct. Option (c) is based on the misconception that the direction of the applied force dictates the direction of the force that the slotted member exerts in turn on the pin. Option (d) falsely takes the force to be parallel to the slot (parallel to the relative motion between the slot and the pin). Option (e) has the force acting in the correct direction, but also features a couple as well. This option may be tempting since the applied force produces a moment about the pin in the indicated direction; it draws on the misconception that a moment due to a force is tantamount to a couple being exerted on the pin.

Sample question on trade-off between equilibrium and the upper limit on the friction force

Two blocks are stacked on a table. The friction coefficient between the blocks and between the lower block and the table is 0.2. (Take this to be both the static and kinetic coefficient of friction). Then, the horizontal 10 N force is applied to the lower block. (N* denotes newtons.)*



What is the horizontal component of the force exerted by the table on the lower block?

(a) 4 N* (b) 6 N* (c) 8 N* (d) 10 N* (e) 18 N*

Figure 4. Example of concept question addressing trade off between equilibrium and limit of friction.

In such problems, one needs to be cognizant both of the forces that would be necessary to maintain equilibrium and of the limits that friction might place on the magnitude of the forces. Students have a tendency to make two errors: either to presume the tangential force is automatically μN (N is normal force) or that the tangential force is the difference between the driving force and what friction (μN) takes away. We do not know the origin of the second misconception, although it may be tied to the idea that friction is not the force between bodies, but is due to the roughness. In any event, option (a) would be arrived at if the tangential force is the difference and if N is falsely taken to be only 30 N*. Option (b) falsely takes the tangential force to be μN, and moreover takes N wrongly to be 30 N*. Option (c) presumes the tangential force is the difference, but with N correctly set to 90 N*. Option (d) is correct as it balances 10 N* and is satisfies the friction condition (tangential force < μN). Option (e) takes the tangential force to be μN, but at least with N correctly set to 90 N*.

Sample question on equilibrium

The bar is maintained in equilibrium by a hand gripping the right end (which is not shown). A positive upward force is applied to the left end. Neglect the force of gravity.

Which of the following could represent the load(s) exerted by the hand?

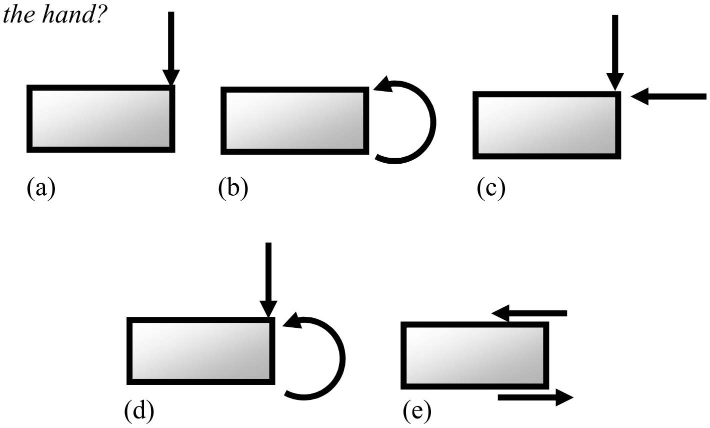


Figure 5. Example of concept question equilibrium conditions.

In maintaining the bar in equilibrium, the hand gripping the right end must keep both the summation of moments and the summations of forces in all directions equal to zero. Option (a), while balancing forces, does not balance moments. Option (b) does not balancing forces, but seems to balance moments (though it does not). Option (c) seems to balance moments (say about the lower right corner) by introducing the horizontal force; however, now horizontal forces do not balance (and moments cannot be balanced about all points). Option (d) is correct. Option (e), like option (b), fails to balance the vertical force.

STATICS CONCEPT INVENTORY

A test comprising a set of 27 questions was devised with the following numbers of questions: free body diagrams (5), static equivalence (3), force at connections (12), friction limit (3), equilibrium (4). The group of questions addressing forces at connections comprises four groups of three questions each; these questions touch on forces on rollers, forces between pins and slotted members, forces on two-force members and forces at general pin joints. Thus, the inventory assesses a total of eight concepts. This test is referred to here as the Statics Concept Inventory or just the inventory. Questions within a given category often have wrong answers that share common misconceptions. Thus, ultimately, not only might one conclude that a student has trouble with free body diagrams, but that the misconceptions or errors tend to be consistently of one type.

The inventory was taken using pencil and paper by students in the mechanical engineering department at Carnegie Mellon University at the start and end of Statics. These students took the test individually during a 50 minute class period, and did not collaborate. (The CM students had had a 3 week exposure to Statics in a freshman mechanical engineering course.) The inventory was also taken by students at the end of Statics at four other universities. (These universities ranged from a local commuter school to an elite research university.) These students were asked by their instructors to take the test by computer outside of class in a time period of approximately one week. (Students downloaded a pdf file of the test, and then entered answers either by filling in a prepared Excel spreadsheet or preparing an email in a specified format, which were then sent electronically to Carnegie Mellon for processing.) Thus, it was impossible to monitor the time student spent taking the test, or whether they had help while doing the test. Students received credit for taking the test, although not for their particular scores.

PSYCHOMETRICS

The first set of statistics pertains to the results for 245 students from five universities taking the test at the end of a statics course. In the combined sample, 77% (n=189) were male and 23% (n=56) were female. The racial/ethnic breakdown was 80% (n=196) white, 2% (n=5) black or African American, 13% (n=32) Asian, 2% (n=4) were Hispanic, and 3% (n=8) were from other racial/ethnic backgrounds. The mean score for the overall sample was 15.71 with a standard deviation of 6.56. An analysis of variance using gender and race/ethnicity as factors was performed to examine differences between total test score means. There was no significant interaction effect between race and gender [$F(4,235)=.417$, $p=.796$], nor were significant main effects of gender [$F(1,235)=.426$, $p=.515$] or race/ethnicity [$F(4,235)=1.578$, $p=.181$] observed. The lack of statistical significance between total test scores indicates that, statistically, the means between groups are equal.

Table 1: Demographic breakdown of respondents.

Gender/Ethnicity	Number	Mean	SD
Male	189	16.37	6.70
Female	56	13.50	5.55
White	196	15.64	6.53
Black	5	16.20	6.57
Asian	32	17.50	5.89
Hispanic	4	16.00	9.63
Other	8	9.88	6.20
Total	245	15.71	6.56

Table 2: ANOVA table for demographic and race of respondent.

Source	*df*	MS	F	*p*
Gender	1	17.493	0.426	.515
Race	4	64.835	1.578	.181
Gender X Race	4	17.143	0.417	.796
Error	235	41.087		

ITEM ANALYSIS

Another important measure is the difficulty of various questions; this is captured by the difficulty index, which is merely the fraction of students who answer the given question correctly [12]. Thus, higher values of the difficulty index correspond to easier questions. In Figure 6, we display the difficulty of questions, which ranged from a low of 0.31 to a high of 0.85. Ideally, the test should be such that significant gains over the semester could be observed, and that students having significantly different levels of conceptual understanding be distinguishable.

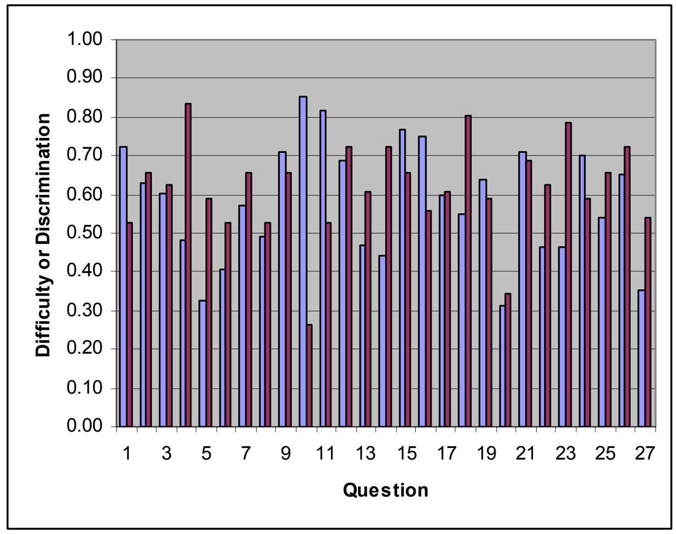


Figure 6. Difficulty (left, lighter) and Discrimination (right, darker) indices for items on SCI.

The discrimination index, which can range from -1 to +1, captures how well students whose overall scores put them in the top third of the sample performed on any particular question in comparison with students in the bottom third [12]. The closer the value is to 1, the more the test distinguishes between students, and this is desirable. The discrimination index is also shown in Figure 6 for each question. The discrimination index ranged from a low of 0.26 to a high of 0.84. In general, one seeks to have the discrimination index for all questions exceed 0.3. With one exception all items had discrimination indices above 0.3. A vry low discrimination index may be indicative of a poorly worded question.

RELIABILITY AND VALIDITY

A test or instrument is evaluated based on two major factors, reliability and validity. Reliability refers to the consistency of test scores over repeated administrations. An estimate of reliability helps to determine how much of the variability in a test is due to measurement error. Validity, on the other hand, refers to whether or not the instrument measures what it was intended to measure. While reliability is rather straightforward to assess, validity is somewhat more difficult. There are three main types of validity; content validity, construct validity, and criterion-related validity. Assessing more levels of validation helps to establish greater evidence of validity.

Reliability
Reliability can be measured using Cronbach's alpha reliability coefficient which is a rating of reliability on a scale from 0 to 1.0. A high coefficient is evidence of good test reliability.

Cronbach's alpha reliability coefficient for the inventory was found to be 0.89, which is strong evidence of reliability.

Content validity
Content validity refers to the ability of the test items to represent the domain of interest. Each of the questions in the inventory focuses on a major conceptual task faced in Statics, and the distracters (wrong answers) were devised to single out distinct errors made by students which could have a conceptual basis. These errors, which were organized into categories [9], were arrived at through several means, including the experience of the first author as an instructor and those of colleagues at two universities, also long time instructors of Statics. Errors were also based on extensive analysis of written solutions to Statics problems requiring the use of multiple Statics concepts. Examples of errors from those solutions were reported earlier [9], along with the above concept organization. Some of those solutions were from students just beginning a Statics course (who had some prior experience with Statics in a freshman engineering course). Solutions to a second set of problems that were analyzed were from students who had completed a Statics course, and therefore displayed conceptual errors which persist after a full semester of instruction in Statics. The analysis of this latter set of problems was conducted by the first author as part of a comparison between performance on an earlier version of this inventory and other measures of performance in Statics [11]. Similar types of errors were observed in these comparison problems as are addressed in the inventory.

Criterion-Related Validity
Criterion-related validity refers to the level of agreement between the test score and an external performance measure. Predictive validity is a type of criterion-related validity that refers to the ability of test scores to be predictive of future criterion measures.

The underlying theoretical construct of this instrument is "Statics conceptual knowledge". If the test does indeed measure statics conceptual knowledge, then it should correlate well with an independent measure of statics knowledge. While there is no other measure specifically of conceptual knowledge in Statics (which is why this effort has been undertaken), an understanding of concepts should be at least partially predictive of overall success in a Statics course. To that end, we compared grades of CMU students in the Statics course with the performance on the inventory. The results are shown in Table 3.

A Spearman's rho (*P*) correlation coefficient computed between inventory score and course grade indicates a high level of association between the two (*P*=-.547, n=105, *p*<.001). The rho statistic is a measure of association between -1.0 to 1.0, with results closer to the absolute value of 1 denoting a stronger relationship between variables. Course grade is an ordinal variable defined as follows: A=1, B=2, and C=3. The negative association (*P* = -.547) indicates that as the inventory total score increases, the Course Grade variable decreases. In other words, as inventory score goes up, it is more likely that grades are 'As' instead of 'Cs'. The strong and statistically significant measure of association is evidence that the total score of the inventory is a valid measure of "statics conceptual knowledge".

More recent comparisons for other universities to be reported also show significant positive correlations between inventory scores and performance on exams in a Statics class.

Construct Validity
Construct validity refers to how well items measure the underlying theoretical construct of the instrument. Do items factor together in a logical, clean manner that is predicted in the theory underlying the development of the test?

To answer this question, a confirmatory factor analysis (CFA) was computed using LISREL 8.54. CFA allows the instrument developer to describe a factor structure and then test the model against the actual data. In the case of the inventory, an eight factor model was tested. The eight factors, or subscales, are Free Body Diagrams, Static Equivalence, Rollers, Two-Force Members, Pin-in-Slot Joint, General Pin Joint, Friction, and Equilibrium.

Table 3: Statics course grade compared with total score on the Statics Inventory.

Statics Inventory Score	Course Grade 'A'	Course Grade 'B'	Course Grade 'C'	Total
10			1	1
11		1		1
12			1	1
13		2	1	3
14			1	1
15	1	3	2	5
16		1	2	3
17	1	3	3	7
18		4	1	5
19	2	4	2	8
20	5	6	3	14
21	2	6		8
22	7	5		12
23	6	4	1	11
24	4	2	1	7
25	9	1		10
26	5	3		8
Total	42	44	19	105

Model fit determines how well the model fits the observed data and can be assessed in a number of ways [13]. For the inventory CFA, model fit was assessed using the chi-square statistic, the Goodness of Fit Index (GFI), the Comparative Fit Index (CFI), and the root mean square approximation (RMSEA). Traditionally, model fit is assessed by comparing the actual data matrix to the reproduced model data matrix using the chi square statistic. If the chi-square statistic is significant, then the model and actual data are significantly different from one another and it can be said that the model does not fit the data well. A non-significant chi-square statistic may indicate a good fitting model; however, since the chi-square statistic is strongly affected by sample size, other indices of model fit are used to assess fit. GFI is a measure of the relative amount of the observed variances and covariances accounted for by the current model. A value of .90 or greater indicates that the model fits the data well. CFI measures improvement of the fit of the model in comparison to a null model. A CFI value of .90 or greater indicates a well defined model. The RMSEA values should be less than .05 for a close model fit, and between .05 and .08 for an acceptable fit.

The inventory model of eight factors had a chi-square value of 314.82 (df=296, *p*=0.22), a GFI value of .90, a CFI value of .91, and a RMSEA value of .067. The chi-square value indicates that the reproduced matrix does not deviate significantly from the original matrix. GFI and CFI values are at the bottom of the acceptable ranges, and the RMSEA value is in the acceptable range. While these values all indicate that the theoretical factor structure is acceptable, there does seem to be room for improvement. Further analysis of individual items should lead to a better factor structure; however, the current factor structure as analyzed by confirmatory factor analysis is acceptable.

PRE-POST CHANGES IN PERFORMANCE

Key statistics from the pre- and post-tests at Carnegie Mellon University are given in Table 4.

Table 4: Pre- and post test scores on Statics Concept Inventory at Carnegie Mellon University.

Test	N	Mean	S.D.	Max.	Min.	Median
Pre	127	10.6	4.1	22	2	11
Post	105	20.34	3.5	26	10	20

In addition to the means, the fraction of correct answers increased from the pre-test to the post-test for every question in the inventory. One can see that performance on this test clearly changed over the course of the semester. (A T-test which looks at whether the statistical difference between pre- and post-test scores could be associated with random variation, yielded a value of $t=-23.886$, for 104 degrees of freedom, or $p<.001$. In other words, the probability is less than 0.001 that the difference in the pre- and post-test scores could be attributed to random variation. Thus, this test does appear to measure an ability which can change (markedly) with a semester studying Statics, affirming its use as a tool to capture gain in conceptual understanding.

INFERENCES REGARDING CONCEPTS THAT STUDENTS FIND PARTICULARLY CHALLENGING

By studying the wrong answers that were chosen for various questions, we can learn about common misconceptions that persist even after instruction. Here we only consider the implications of the fraction of total students choosing each answer.

As pointed out early, even after instruction, many students answered questions addressing the limits of friction incorrectly. Although all wrong answers were chosen (in significant numbers by the lower third of scorers), the most commonly chosen answer corresponded to the assumption that the friction force equals μN, with N computed correctly (option (e) in Figure 4). In fact, the lower third of scorers chose this option more often than the correct answer for two out of three friction problems.

Questions addressing static equivalence (e.g., Figure 2 above) were also found to be answered incorrectly by many students. All wrong answers save one were chosen by many students. In two out of three such problems, students chose three wrong answers more often than the correct answer. Both major types of errors - having the net force be inconsistent with the original and having the net moment inconsistent with the original – were made by many students. Likewise, in questions addressing equilibrium (e.g., Figure 5 above), one could see many examples in which answers indicated a neglect of force equilibrium and many other examples of answers indicating a neglect of moment equilibrium.

In the questions addressing free body diagrams (e.g., Figure 1 above), by far the most commonly chosen incorrect answers were those with an internal force inappropriately placed in the diagram. Option (c) in Figure 1 is an example of an internal force.

In the remaining 12 questions addressing various connections, for example rollers and slotted members (Figure 3 above), one finds a variety of wrong answers. However, it is quite common for students to presume that a force acts in directions that are parallel to or perpendicular to one of the members, even if that has nothing to do with the actual direction. Students also tend often to make a choice which is consistent with the force apparently balancing an applied force, again even if the force locally cannot act in that direction.

For each of the question types in the inventory, one can envision more detailed analyses that focus on patterns of errors of individual students. As an example, in equilibrium questions, some students might tend to choose answers in which force equilibrium is violated, while others where only moment equilibrium is violated. There are such distinct types of errors in all questions. Such fine-grained information on student thinking, if made available to instructor and student, could lead to more targeted remedial instruction. Such analyses will be undertaken and reported in the future.

POTENTIAL USES OF CONCEPT INVENTORIES

Concept inventories may be used to improve student learning in many ways, a few of which are pointed out here. When administered at the end of a course, the inventory can provide the instructor with feedback on those concepts that may need more attention in the future. Or, since most of the concepts may have already been covered by, say, two-thirds through the course, the test could be administered at that point. If the results could be analyzed rapidly and provide diagnoses as to conceptual lapses, then remedial exercises might be tailored to address these lapses. An inventory could also be used at the start of a follow-on course (e.g., dynamics or mechanics of materials), to provide instructors with a picture of the starting knowledge of their students. Finally, the questions themselves might stimulate ideas for instruction that is more conceptually based or might suggest in-class assessment exercises.

SUMMARY AND CONCLUSIONS

This paper has presented a new instrument for assessing understanding of concepts of Statics. The Statics Concept Inventory features multiple choice questions that reflect a conceptual framework articulated previously; wrong answer options (distractors) reflect misconceptions and errors commonly observed in students work. The test comprises 27 such questions, addressing free body diagrams, static equivalence, equilibrium, forces at connections and friction. This test was administered to students before and after a Statics course at Carnegie Mellon University, and to students at the end of Statics courses at four other universities. Psychometrics based on the sample of 245 test-takers indicated that the inventory offers reliable and valid measures of conceptual knowledge in Statics. On the basis of this test, one can infer which concepts students in general tend to have the most difficulties with, as well as the misconceptions that appear to be most prevalent. Larger numbers of students have now taken the Statics Concept Inventory. Instructors who are interested in having their students take the test can contact the first author.

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